[2] $y_2 = ve^{-2x}$

$y_2' = v'e^{-2x} - 2ve^{-2x}$

$y_2'' = v''e^{-2x} - 2v'e^{-2x}$
$\quad - 2v'e^{-2x} + 4ve^{-2x}$
$= v''e^{-2x} - 4v'e^{-2x} + 4ve^{-2x}$

$xy_2'' + (4x+4)y_2' + (4x+8)y_2$

$$= e^{-2x}\big(xv'' - 4xv' + 4xv + (4x+4)v' + (-8x-8)v + (4x+8)v \big)$$ (1)

$$= \underline{e^{-2x}(xv'' + 4v')}$$ MANDATORY CHECKPOINT: NO V WITHOUT ' ($\frac{1}{2}$)

$= 0 \longrightarrow xv'' + 4v' = 0$ LET $u = v'$

$\underline{xu' + 4u = 0}$ ($\frac{1}{2}$)

$x\frac{du}{dx} = -4u$

$\underline{\int \frac{1}{u}\,du = \int -\frac{4}{x}\,dx}$ MANDATORY CHECKPOINT: SEPARABLE ($\frac{1}{2}$)

$\ln|u| = -4\ln|x|$

$v' = \underline{u = x^{-4}}$ ($\frac{1}{2}$)

$\underline{v = -\frac{1}{3}x^{-3}}$ ($\frac{1}{2}$)

$y_2 = x^{-3}e^{-2x}$

$\underline{y = c_1 e^{-2x} + c_2 x^{-3}e^{-2x}}$ ($\frac{1}{2}$)

[3] $2r^2+(4-2)r+5=0$

$$2r^2+2r+5=0 \longrightarrow r=\frac{-2\pm\sqrt{4-40}}{4}=\frac{-2\pm 6i}{4}=-\frac{1}{2}\pm\frac{3}{2}i \quad \frac{1}{2}$$

$$x=c_1t^{-\frac{1}{2}}\cos\tfrac{3}{2}\ln t+c_2t^{-\frac{1}{2}}\sin\tfrac{3}{2}\ln t \quad \frac{1}{2}$$

$$x'=\boxed{\begin{aligned}&-\tfrac{1}{2}c_1t^{-\frac{3}{2}}\cos\tfrac{3}{2}\ln t-\tfrac{3}{2}c_1t^{-\frac{3}{2}}\sin\tfrac{3}{2}\ln t\\&+\tfrac{3}{2}c_2t^{-\frac{3}{2}}\cos\tfrac{3}{2}\ln t-\tfrac{1}{2}c_2t^{-\frac{3}{2}}\sin\tfrac{3}{2}\ln t\end{aligned}} \quad 1$$

$x(1)=c_1=2$

$$x'(1)=-\tfrac{1}{2}c_1+\tfrac{3}{2}c_2=-5 \longrightarrow -1+\tfrac{3}{2}c_2=-5 \longrightarrow c_2=-\tfrac{8}{3} \quad \frac{1}{2}$$

$$x=2t^{-\frac{1}{2}}\cos\tfrac{3}{2}\ln t-\tfrac{8}{3}t^{-\frac{1}{2}}\sin\tfrac{3}{2}\ln t \quad \frac{1}{2}$$

[4] $4r^4+4r^3+r^2-6r+2=0$

$$\begin{array}{r|rrrrr}
\frac{1}{2} & 4 & 4 & 1 & -6 & 2 \\
 & & 2 & 3 & 2 & -2 \\
\hline
\frac{1}{2} & 4 & 6 & 4 & -4 & \boxed{0} \\
 & & 2 & 4 & 4 & \\
\hline
 & 4 & 8 & 8 & \boxed{0} &
\end{array}$$

$\frac{1}{2}$ $\frac{1}{2}$

$(r-\frac{1}{2})^2(4r^2+8r+8)=0$

$4(r-\frac{1}{2})^2(r^2+2r+2)=0$

$$r=\frac{-2\pm\sqrt{4-8}}{2}=\frac{-2\pm 2i}{2}=\underline{-1\pm i}$$ $\frac{1}{2}$

$y_h=\underline{c_1e^{\frac{1}{2}x}+c_2xe^{\frac{1}{2}x}+c_3e^{-x}\cos x+c_4e^{-x}\sin x}$ $\frac{1}{2}$

$y_p=\underline{(Ae^{-x}\cos x+Be^{-x}\sin x)x}+\underline{Dx^3+Ex^2+Fx+G}$ $\frac{1}{2}$ $\frac{1}{2}$

$\underline{+\left[(Hx^2+Jx+K)e^{\frac{1}{2}x}\right]x^2}$ 1

[5] $r^2+4=0 \longrightarrow r=\pm 2i$ $\frac{1}{2}$ $\longrightarrow y_h = c_1\cos 2x + c_2\sin 2x$ $\frac{1}{2}$

$$y_p = ((Ax+B)\cos 2x + (Cx+D)\sin 2x)x + Ee^{-2x}$$

$$= (Ax^2+Bx)\cos 2x + (Cx^2+Dx)\sin 2x + Ee^{-2x}$$ 1

$$y_p' = (2Ax+B)\cos 2x + (-2Ax^2-2Bx)\sin 2x$$
$$+ (2Cx^2+2Dx)\cos 2x + (2Cx+D)\sin 2x - 2Ee^{-2x}$$

1 $$= \boxed{(2Cx^2+(2A+2D)x+B)\cos 2x + (-2Ax^2+(-2B+2C)x+D)\sin 2x - 2Ee^{-2x}}$$

$$y_p'' = (4Cx+(2A+2D))\cos 2x + (-4Cx^2+(-4A-4D)x-2B)\sin 2x$$
$$+ (-4Ax^2+(-4B+4C)x+2D)\cos 2x + (-4Ax+(-2B+2C))\sin 2x$$
$$+4Ee^{-2x}$$

1 $$y_p'' = \boxed{(-4Ax^2+(-4B+8C)x+(2A+4D))\cos 2x + (-4Cx^2+(-8A-4D)x+(-4B+2C))\sin 2x + 4Ee^{-2x}}$$

$+4y_p$ $$+(4Ax^2+4Bx)\cos 2x + (4Cx^2+4Dx)\sin 2x + 4Ee^{-2x}$$

$$= (8Cx+(2A+4D))\cos 2x + (-8Ax+(-4B+2C))\sin 2x + 8Ee^{-2x}$$ 1

$$= 5\cos 2x - 4x\sin 2x + 3e^{-2x}$$

$8C=0 \to C=0$ $\quad -8A=-4 \to A=\frac{1}{2}$ $\quad 8E=3$ $\frac{1}{2}$

$E=\frac{3}{8}$

$\frac{1}{2}$ $2A+4D=5$ $\quad -4B+2C=0$

$D=\frac{1}{4}(5-2A)=1$ $\quad B=-\frac{1}{2}C=0$

$$y_p = \frac{1}{2}x^2\cos 2x + x\sin 2x + \frac{3}{8}e^{-2x}$$

$$y = \frac{1}{2}x^2\cos 2x + x\sin 2x + \frac{3}{8}e^{-2x} + c_1\cos 2x + c_2\sin 2x$$ 1